II. *The Bakerian Lecture on the Force of Percussion. By* William Hyde Wollaston, *M. D. Sec. R. S.*

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WHEN different bodies move with the same velocity, it is universally agreed that the forces, which they can exert against any obstacle opposed to them, are in proportion to the quantities of matter contained in the bodies respectively. But, when equal bodies move with unequal velocities, the estimation of their forces has been a subject of dispute between different classes of philosophers. LEIBNITZ and his followers have maintained that the forces of bodies are as the masses multiplied into the *squares* of their velocities, (a multiple to which I shall for conciseness give the name of *impetus);* while those, who are considered as NEWTONIANS, conceive that the forces are in the *simple ratio* of the velocities, and consequently as the *momentum* or *quantitas motus*, a name given by NEWTON to the multiple of the velocity of a body simply taken into its quantity of matter.

It cannot be expected that at this time any new experiment should be thought of, by which the controversy can be decided, since the most simple experiments that have already been appealed to by either party have received different interpretations from their opponents, although the facts were admitted.

My object in the present Lecture is to consider which of these opinions respecting the force exerted by moving bodies is most conformable to the usual meaning of that word, and to shew that the explanation given by NEWTON of the third law of motion is in no respect favourable to those who in their view of this question have been called NEWTONIANS.

If bodies were made to act upon each other under the circumstances which I am about to describe, the leading phænomena would occur, which afford the grounds of reasoning on either side.

Let a ball of clay or of any other soft and wholly inelastic substance be suspended at rest, but free to move in any direction with the slightest impulse; and let there be two pegs similar and equal in every respect inserted slightly into its opposite sides. Let there be also two other bodies, A and B, of any magnitude, which are to each other in the proportion of 2 to 1; suspended in such a position, that when perfectly at rest they shall be in contact with the extremities of the opposite pegs without pressing against them. Now if these bodies were made to swing with motions so adapted that in falling from heights in the proportion of 1 to 4 they might strike at the same instant against the pegs opposite to them, the ball of clay would not be moved from its place to either side; nevertheless the peg impelled by the smaller body B, which has the double velocity, would be found to have penetrated twice as far as the peg impelled by A.

It is unnecessary to make the experiment precisely as here stated, since the results are admitted as facts by both parties; but upon these facts they reason differently.

One side observing that the ball of clay remains unmoved, considers the proof indisputable that the action of the body A is equal to that of B, and that their forces are properly measured by their momenta, which are equal, because their velocities are in the simple inverse ratio of the bodies. Their opponents think it equally proved by the unequal depths to which the pegs have penetrated, that the causes of these effects are unequal, as they find to be the case in their estimation of the forces by the squares of the velocities.

One party is satisfied that equal *momenta* can resist equal pressures during the same *time;* the other party attend to the *spaces* through which the same moving force is exerted, and finding them in the proportion of 2 to 1, are convinced that the *vis viva* of a body in motion is justly estimated by its magnitude and the square of its velocity jointly.

The former conception of a quantity dependent on the continuance of a given *vis motrix* for a certain *time* may have its use, when correctly applied, in certain philosophical considerations; but the latter idea of a quantity resulting from the same force exerted through a determinate *space* is of greater practical utility, as it occurs daily in the usual occupations of men; since any quantity of work performed is always appreciated by the extent of effect resulting from their exertions; for it is well known, that the raising any great weight 40 feet would require 4 times as much labour as would be requisite to raise an equal weight to the height of 10 feet, and that in its slow descent the former would produce 4 times the effect of the latter in continuing the motion of any kind of machine. Moreover, if the weights so raised were suffered to fall freely through the heights that have been ascended by means of 4 and of 1 minute's labour, the velocities acquired would be in the ratio of 2 to 1, and the squares of the velocities in proportion to the quantities of labour from which they originated, or as 4 to 1; and if the forces acquired by their descent were employed in driving piles, their more sudden effects produced would be found to be in that same ratio.

This species of force has been, first by BERNOUILLI and afterwards by SMEATON, very aptly denominated mechanic force; and when by force of percussion is meant the quantity of mechanic force possessed by a body in motion, to be estimated by its quantity of mechanic effect, I apprehend it cannot be controverted that it is in proportion to the magnitude of the body and to the square of its velocity jointly.

But of this quantity of force NEWTON no where treats, and has accordingly given no definition of it. If, after defining what he meant by the *quantitas acceleratrix*, and *quantitas motrix*, he had had occasion to convey an equally distinct idea of the *quantitas mechanica* resulting from the continued action of any force, he might, not improbably, have proceeded conformably to the definition given by SMEATON, and have added

——quantitas mechanica est mensura proportionalis spatio per quod data vis motrix exercetur;

or, if speaking with reference to the accumulated energy communicated to a body in motion,

———proportionalis quadrato velocitatis quam in dato corpore generat.

But, if we attend to the words of his preface to the first edition of his *Principia*, he evidently had no need of such a definition;

"Nos autem non artibus sed philosophiæ consulentes, deque "potentiis non manualibus sed naturalibus scribentes," &c.

And again, nearly to the same effect in the *Scholium*, which follows the laws of motion, "Cæterum mechanicam tractare "non est hujus instituti."

In the third law of motion he has on the contrary been supposed to speak of this force from an ambiguity in the signification of the words *actio* and *reactio*. By these, however, NEWTON certainly meant a mere *vis motrix* or pressure, as he himself explains them. "Quicquid premit vel trahit alterum, "tantundem ab eo premitur vel trahitur. Si quis lapidem "digito premit, premitur et hujus digitus a lapide," &c. The same meaning is equally evident from his demonstration of the third corollary to the laws, in which he asserts that the *quantitas motûs* of two or more bodies estimated in any given direction is not altered by their action upon each other. The demonstration begins thus:

"Etenim actio eique contraria reactio æquales sunt per legem "tertiam, ideoque per legem secundam æquales in motibus "efficient mutationes versus contrarias partes." Now, if he had considered the third law as implying equality of more than mere moving forces, there could have been no occasion to refer to the second law, with a view thence to deduce the equality of momenta produced.

Some authors however have interpreted the third law differently, and accordingly have expressed a difficulty in comprehending the simple illustration given by NEWTON. When they say that action is equal to reaction, they mean not only that the instantaneous intensity of the moving forces, or pressures opposed to each other, are necessarily equal, but conceive also a species of accumulated force residing in a moving body, which is capable of resisting pressure during a *time* that is proportional to its momentum or quantitas motus.

If it be of any real utility to give the name of force to this complex idea of *vis motrix* extended through time, as well as that of *momentum* to its effects when unresisted, it would be requisite to distinguish this force always by some such appellation as *momental* force; for it is to be apprehended that for want of this distinction many writers themselves, and it is certain that many readers of diquisitions on this subject have confounded and compared together *vis motrix*, *momentum*, and *vis mechanica:* quantities, that are all of them totally dissimilar, and bear no more comparison to each other, than lines to surfaces, or surfaces to solids.

In practical mechanics, however, it is at least very rarely that the *momentum* of bodies is in any degree an object of consideration: the strength of machinery being in every case to be adapted to the *quantitas motrix*, and the extent and value of the effect to be produced depending upon the *quantitas mechanica* of the force applied, or in other words to the space through which a given vis motrix is exerted.

The comparative velocities given by different quantities of mechanic force to bodies of equal or unequal magnitude have been so distinctly treated of by SMEATON, in a series of most direct experiments,* that it would be a needless waste of time to reconsider them in this place. So also, on the contrary, the quantities of extended mechanic effect producible by bodies moving with different quantities of impetus have been as clearly traced by the same accurate experimentalist.†

But there is one view, in which the comparative forces of

* Phil. Trans. Vol. LXVI. 450.

† Vol. LXXII. 337.

impact of different bodies was not examined by SMEATON, and it may be worth while to shew that when the whole energy of a body A is employed without loss in giving velocity to a second body B, the *impetus* which B receives is in all cases equal to that of A, and the force transferred to B, or by it to any third body C, (if also communicated without loss, and duly estimated as a mechanic force,) is always equal to that from which it originated.

As the simplest case of entire transfer, the body A may be supposed to act upon B in a direct line through the medium of a light spring, so contrived that the spring is prevented by a ratchet from returning in the direction towards A, but expands again entirely in the direction towards B, and by that means exerts the whole force which had been wound up by the action of A, in giving motion to B alone. In this case, since the moving force of the spring is the same upon each of the bodies, the accelerating force acting upon B at each point is to the retarding force opposed to A at the corresponding points in the reciprocal ratio of the bodies, and the squares of the velocities produced and destroyed by its action through a given space will consequently be in that same ratio. The momentum, which is in the simple reciprocal ratio of the bodies, might consequently be increased at pleasure by the means proposed, in the subduplicate ratio of the bodies employed; and if momentum were an efficient force capable of reproducing itself, and of overcoming friction in proportion to its estimated magnitude, the additional force acquired by such a means of increase, might be employed for counteracting the usual resistances, and perpetual motion would be easily effected. But since the *impetus* remains unaltered, it is evident that the utmost which the body B could effect in return would be the reproduction of A's velocity, and restitution of its entire mechanic force neither increased nor diminished, excepting by the necessary imperfection of machinery. The possibility of perpetual motion is consequently inconsistent with those principles which measure the quantity of force by the quantity of its extended effect, or by the square of the velocity which it can produce.

In estimating the utmost effect which one body can produce upon another at rest, the same result is obtained by employing *impetus* as ascensional force, according to HUYGENS; for if the body A were allowed to ascend to the height due to its velocity, and if by any simple mechanical contrivance of a lever or otherwise the body B were to be raised by the descent of A, it is well known that the heights of ascent would be reciprocally as the bodies; and consequently that the *square* of the velocity to be acquired by free descent of B would be in that ratio, and the quantity of mechanic force would be preserved as before unaltered.

It may be of use also to consider another application of the same energy, and to shew more generally that the same quantity of total effect would be the consequence not only of direct action of bodies upon each other, but also of their indirect action through the medium of any mechanical advantage or disadvantage; although the time of action might by that means be increased or decreased in any desired proportion. For instance, if the body supposed to be in motion were to act by means of a lever upon a spring placed at a certain distance from the centre of motion, the retarding force opposed to it would be inversely as the distance of the body from the centre; and since the space through which the body would move to lose its whole velocity would be reciprocally as the retarding force, the angular motion of the lever and space through which the spring must bend, would be the same, at whatever point of the lever the body acted. And conversely, the reaction of the spring upon any other body B, would in all positions communicate to it the same velocity.

It may be remarked, however, that the times in which these total effects are produced may be varied at pleasure in proportion to the distances at which the bodies are placed from the centre of motion; and it should not pass unobserved that, although the intensity of any *vis motrix* is increased by being placed at what is called a mechanical advantage, yet on the contrary, any quantity of mechanic force is not liable to either increase or diminution by any such variation in the mode of its application.

Since we can by means of any mechanic force consisting of a *vis motrix* exerted through a given *space*, give motion to a body for the purpose of employing its *impetus* for the production of any sudden effect, or can, on the contrary, occasion a moving body to ascend, and thus resolve its *impetus* into a moving force ready to exert itself through a determinate space of descent, and capable of producing precisely the same quantity of mechanic effect as before, the force depending on *impetus* may justly be said to be of the same kind as any other mechanic force, and they may be strictly compared as to quantity.

In this manner we may even compare the force of a body in motion to the same kind of force contained in a given quantity of gunpowder, and may say that we have the same quantity of mechanic force at command whether we have 1lb. of powder, which by its expansion could give to 1 ton weight a velocity sufficient to raise it through 40 feet, or the weight actually raised to that height and ready to be let down gradually, or the same weight possessing its original velocity to be employed in any sudden exertion.

By making use of the same measure as in the former cases, a distinct expression is likewise obtained for the quantity of mechanic force given to a steam-engine by any quantity of coals; and we are enabled to make a comparison of its effect with the quantity of work that one or more horses may have performed in a day, each being expressed by the space through which a given moving force is exerted. In the case of animal exertion however, considerable uncertainty always prevails in consequence of the unequal powers of animals of the same species, and varying vigour of the same animal. The information which I have received in reply to inquiries respecting the weights raised in one hour by horses in different situations, has varied as far as from 6 to 15 tons to the height of 100 feet. But although the rate at which mechanic force is generated may vary, any quantity of work executed is the same, in whatever time it may have been performed.

In short, whether we are considering the sources of extended exertion or of accumulated energy, whether we compare the accumulated forces themselves by their gradual or by their sudden effects, the idea of mechanic force in practice is always the same, and is proportional to the *space* through which any moving force is exerted or overcome, or to the *square* of the velocity of a body in which such force is accumulated.